Underground water contamination studies of the part of foreground of „Żelazny Most” reservoir using cokriging

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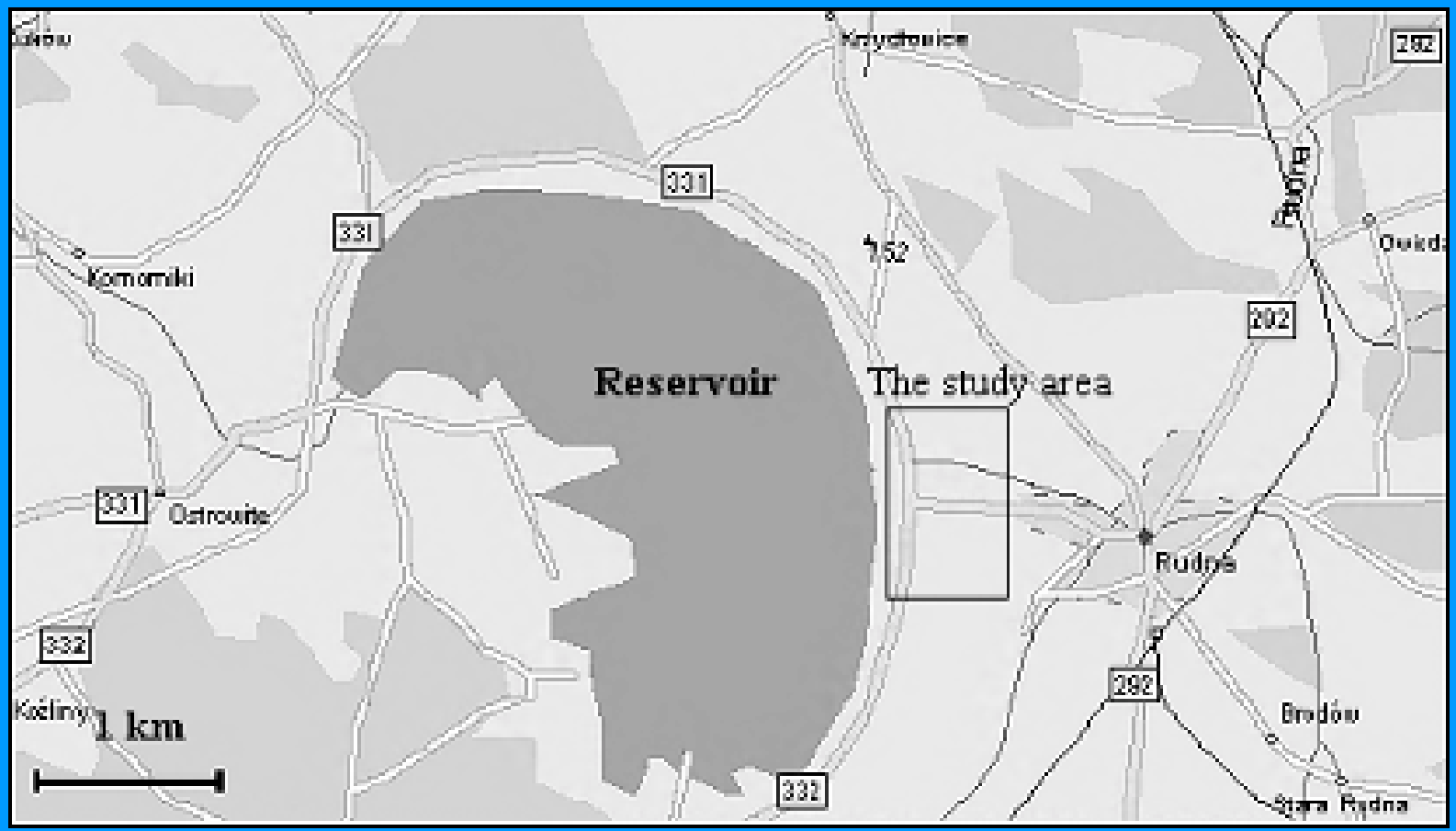
Outline

- “Żelazny Most” reservoir
- Study site
- Data set
- Variograms
- Cokriging
- Results
- Conclusions

“Żelazny Most” reservoir

- *„Żelazny Most” is the biggest reservoir of wastes out of copper mines in Europe.*
- *It is on the list of 80 objects and industrial plants which exert the biggest negative influence on natural environment in Poland.*
- *Reservoir is about 17 km^2 in area.*
- *Protective area around reservoir is about 11 km^2 in area.*
- *Copper mines of KGHM Polska Miedź SA stored there until 2001 $320*10^6$ m^3 wastes and $10*10^6$ m^3 water from mines.*

The map of „Żelazny Most” reservoir location



Study site

The study area is a part of eastern foreground

The choice was based on the available data set so as to the main variable „u” (chemical measurements) was replenished by the secondary variable „v” (geoelectrical measurements).

The chosen site was 1046.0 m long (along the north – south axis) and 695.0 m wide (along the east – west axis).

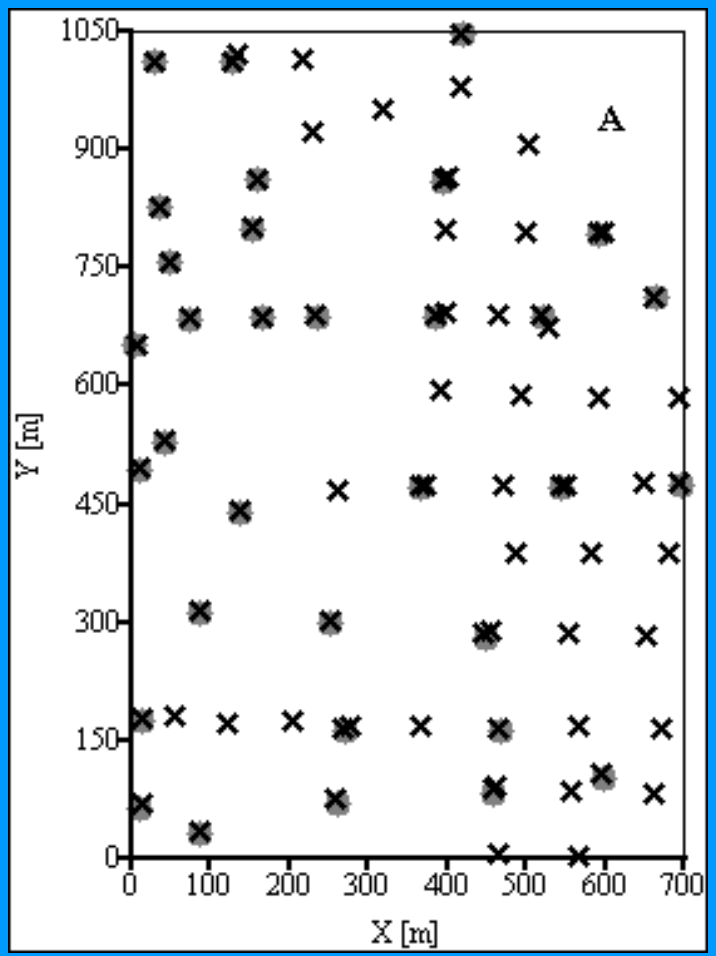
The average distance to the nearest point of the same type was 120.0 m for „u” and 60.0 m for „v”.

Data set

Two kinds of data: chemical and geoelectrical measurements

- *Chemical samples were taken from wells placed on the foreground of the reservoir and they included total mineralization e.g. concentrations of: Cl^-, SO_4^{2-}, Na^+.*
- *Geoelectrical measurements were made at points located on the site but in different places than mineralization measurements. They were made using probing method.*

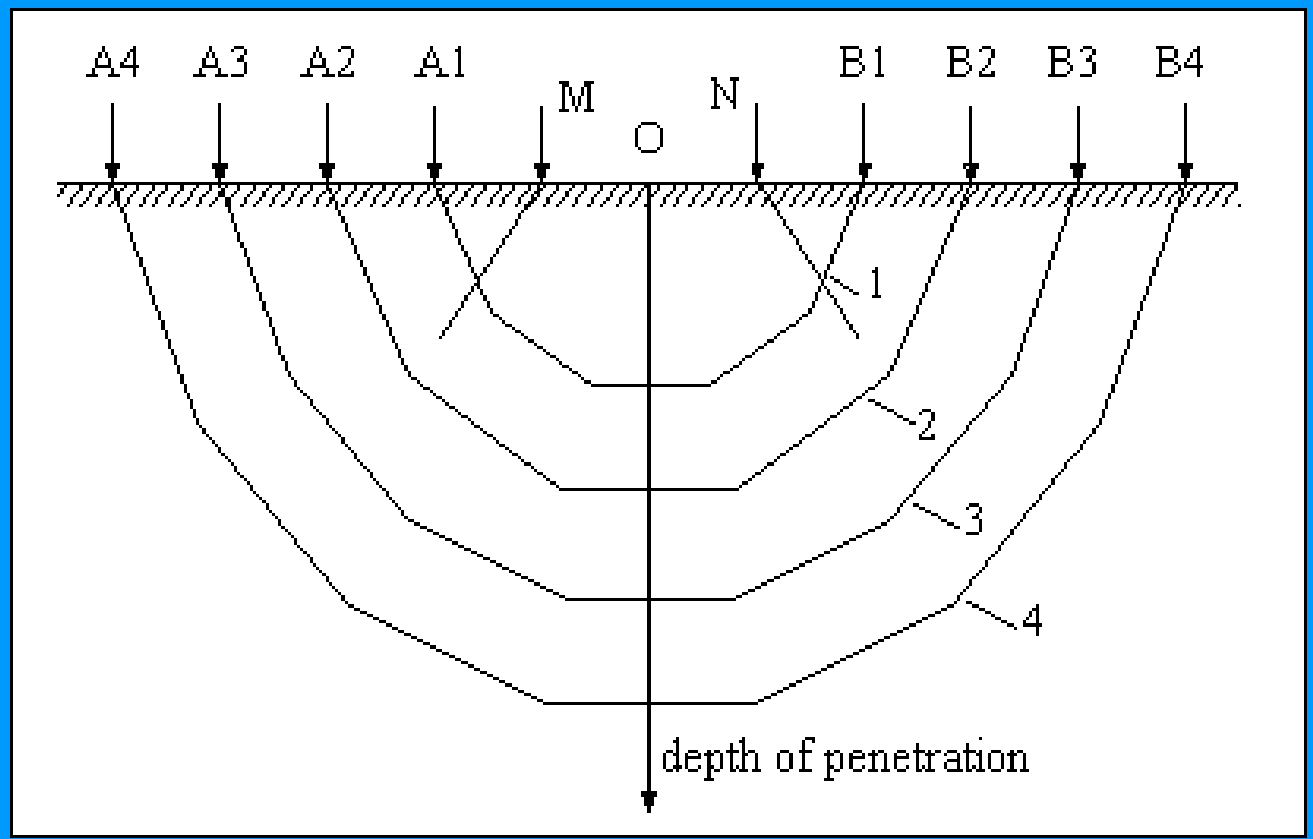
The map of sample points' locations



gray circles denote chemical measurements

black diagonal cross' denote geoelectrical measurements

Scheme of geoelectrical measurement



Descriptive statistics of both variables

	Total mineralisation	Electric resistance
Number	127	67
Mean	3218.4 [mg/l]	50.8 [Ωm]
Median	1040.0 [mg/l]	44.0 [Ωm]
Mode	276.0 [mg/l]	34.0 [Ωm]
Lower quartile	310.0 [mg/l]	34.0 [Ωm]
Upper quartile	5360.0 [mg/l]	60.0 [Ωm]
Minimum	102.0 [mg/l]	17.5 [Ωm]
Maximum	18732.0 [mg/l]	240.0 [Ωm]
Range	18630.0 [mg/l]	222.5 [Ωm]
Variance	$2.230 \cdot 10^7$ $[(mg/l)^2]$	943.8 $[(\Omega m)^2]$
Standard deviation	358.18 [mg/l]	3.75 [Ωm]
Skewness	1.467	3.786

Variograms

Variogram (semivariance) is defined as a half of average squared difference between pairs of sample points:

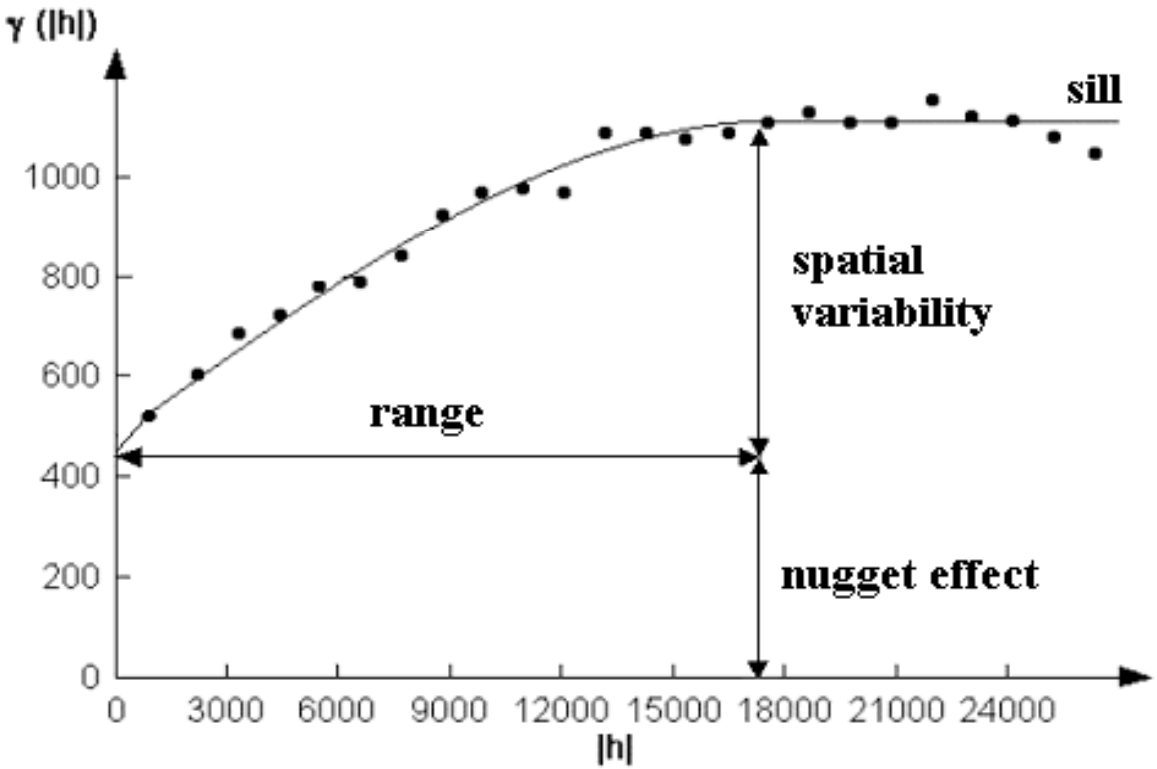
$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{(i,j)\mathbf{h}_{ij}=\mathbf{h}} (v_i - v_j)^2$$

N(h) – number of pairs

h – separating diastance

v – sample value

Variograms


γ (|h|)
sill
1000
800
600
400
200
0
spatial
variability
range
nugget effect
0
3000
6000
9000
12000
15000
18000
21000
24000
|h|

Cokriging

Cokriging is a kind of kriging that estimates main variable „u" based on information bringing by secondary variable „v" considering spatial character of the phenomena.

It finds application when main measurements are difficult to perform, or expensive (too sparse sampled).

Estimated value is calculated by using linear combination of both variables:

$$\hat{u}_0(x_0) = \sum_{i=1}^{n} a_i \cdot u_i(x_i) + \sum_{j=1}^{m} b_j \cdot v_j(x_j)$$

Cokriging

Cokriging can be used when variogram models for γ_u, γ_v and cross-variogram γ_{uv} are known:

$$\gamma_u(h) = u_0\gamma_0(h) + u_1\gamma_1(h) + \ldots + u_k\gamma_k(h)$$

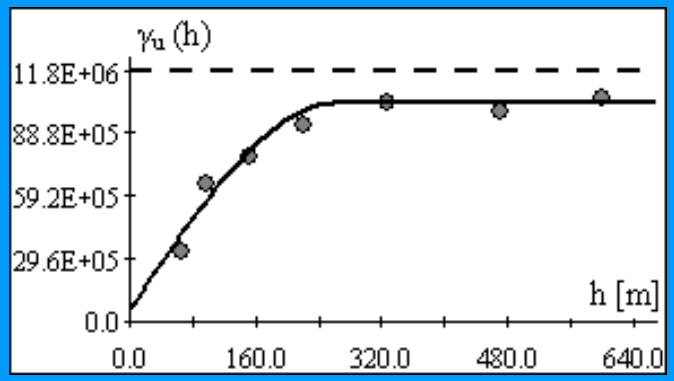
$$\gamma_v(h) = v_0\gamma_0(h) + v_1\gamma_1(h) + \ldots + v_k\gamma_k(h)$$

$$\gamma_{uv}(h) = w_0\gamma_0(h) + w_1\gamma_1(h) + \ldots + w_k\gamma_k(h)$$

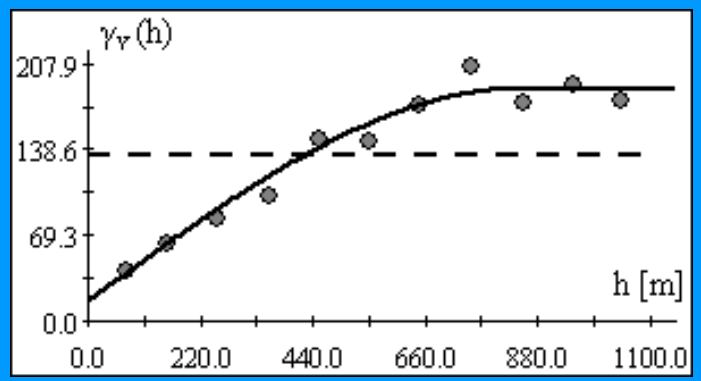
These equations must be positively determined. It is true when the following equations are true:

$$u_i > 0 \wedge v_i > 0 \qquad u_i \cdot v_i > w_i \cdot w_i$$

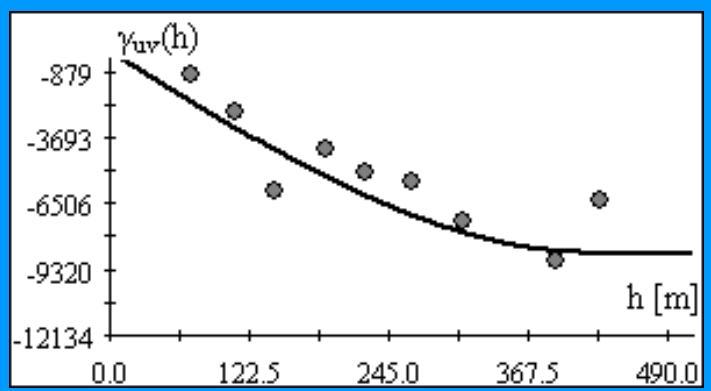
Cokriging



The variogram model for main variable „u



The variogram model for secondary variable „v”



The cross – variogram model

Underground water standards thresholds for mineralization

Total mineralisation [mg/l]	
I class	< 500.0
II class	< 1000.0
III class	< 1200.0
Maximum value in sewage piped away to ground or water	2000.0

Ministry of Environment, 1991.

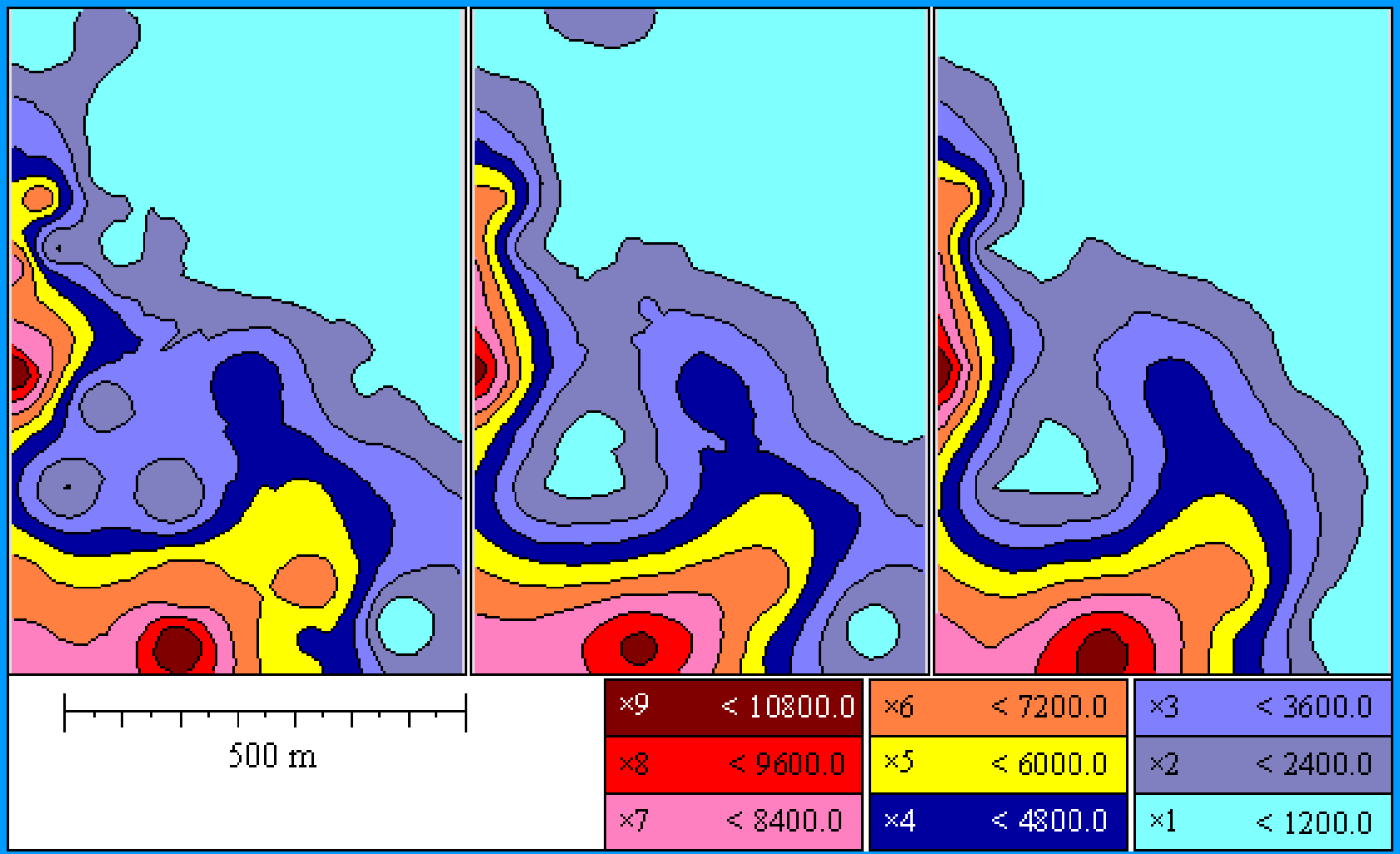
Maps of mineralization

(values expressed in [mg/l])

IDW

Kriging

Cokriging

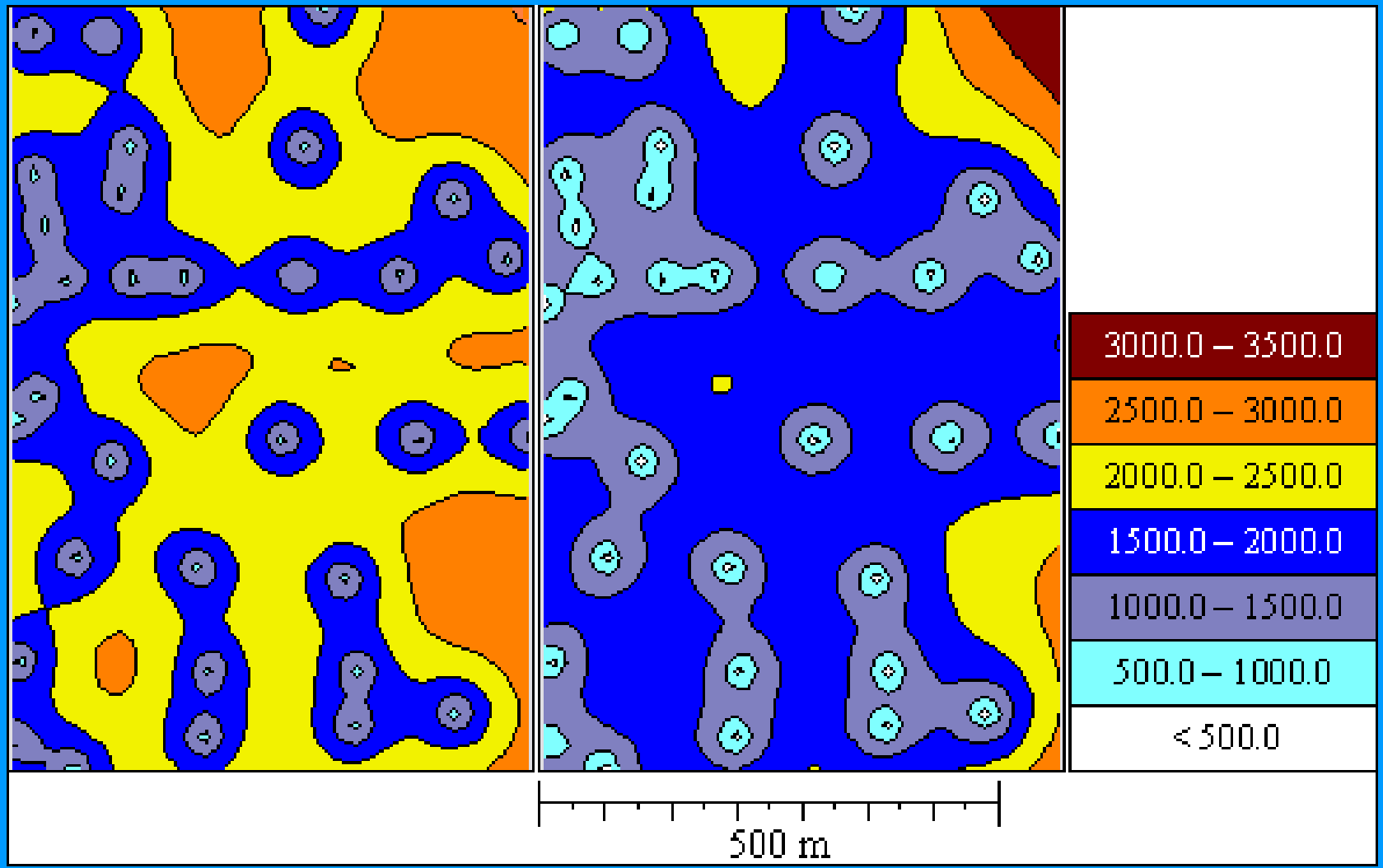


Standard errors maps for mineralization

(values expressed in [mg/l])

Kriging

Cokriging



Results of the cross-validate analysis

Evaluating effective parameters for interpolations was made using cross-validation analysis

- *Each measured point in a spatial domain was individually removed and its value was estimated as though it was never there*
- *Actual and estimated values were compared and correlation coefficient between them was calculated*

	IDW	Kriging	Cokriging
Correlation coeff. between actual and estimated value	0.673	0.747	0.773
Regression coefficient	0.970	1.071	0.976
Standard error of the regression coefficient	0.191	0.171	0.144

Conclusions

- **Cokriging is effective method to integrate two (or more) kinds of spatial information.**
- **Cokriging makes it possible to reduce a significant number of expensive measurements of primary variable.**
- ***Additional information bringing with geoelectrical measurements significantly improved interpolation of the mineralization on the "Żelazny Most" reservoir foreground.***